mode, so that its impulse response is the sum of a ramp function and damped sinusoids (see Ref. 11 for details). Consequently, Wilson's method is not directly applicable. Figure 1 compares the impulse response of the original system with that of the optimal fourth-order model obtained for several values of α. It appears, as expected, that the impulse response is approximated over an interval of duration proportional to $1/\alpha$.

Conclusion

We have shown how methods now existing for optimal model reduction of asymptotically stable sytems can be extended to nonasymptotically-stable systems. This extension can be very simply implemented in software now existing, making it useful and convenient for the optimal model reduction of marginally stable and unstable systems.

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Angular Motion Influence on Re-entry Vehicle Ablation or Erosion Asymmetry Formation

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Nomenclature

a	$=E/2\Omega^2$; $(\theta^2_{max}+\theta^2_{min})/2$
b	$=(E^2-4K^2\Omega^2)^{1/2}/2\Omega^2$; $(\theta^2_{max}-\theta^2_{min})/2$
E	= constant determined by initial conditions, Eq. (6)
I	= pitch or yaw moment of inertia
I_x	= roll moment of inertia
K	= constant determined by initial conditions, Eq. (5)
p	= roll rate
p_r	= roll rate parameter, $\mu p/2$
t	= time
θ	= pitch angle (Euler angle)
θ_{max}	= maximum value of θ during epicyclic oscillation
θ_{min}	= minimum value of θ during epicyclic oscillation
$\dot\theta$	= pitch rate
μ	$=I_x/I$
τ	= phase angle, Eq. (8)
ϕ	= roll angle relative to wind (Euler angle)
$\Delta\phi_p$	$=\pi p_r/\Omega$
$\dot\phi$	= windward-meridian rotation rate
$\dot\phi_{max}$	= maximum value of $\dot\phi$ during epicyclic oscillation
$\dot\phi_{min}$	= minimum value of $\dot\phi$ during epicyclic oscillation
$\dot\psi$	= precession rate
$\dot\Psi$	$=\dot\psi-p_r$
ω	= undamped natural pitch frequency
Ω	$=(\omega^2+p_r^2)^{1/2}$

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Introduction

A BALLISTIC re-entry vehicle usually enters the atmosphere with some angular misalignment between the vehicle's axis of symmetry and its velocity vector which together, by definition, comprise the entry total angle of attack.[1,2] The angle of attack converges with increasing atmospheric density until it reaches a quasisteady trim value determined by the magnitude of mass and configurational asymmetries. During the period in which the angle of attack converges, the motion is generally epicyclic and is characterized by a highly transient windward-meridian rotation behavior, in contrast to trimmed motion in which both the angle of attack and the windward meridian tend to be quasisteady.[3] It has been postulated that the change in the vehicle's shape as a result of combined ablation and erosion should occur preferentially along surface meridians that spend the longest duration windward, i.e., where the windward-meridian rotation rate is minimum. Such points would be subjected to maximum cumulative pressures and heating and, in the case of erosive environments, to maximum cumulative particle impacts.

The coupling between angle of attack and windward-meridian rotation rate is derived for undamped epicyclic motion. The locus of meridians about the vehicle where the windward-meridian rotation rate is minimum and where incipient shape change would be expected to occur is calculated, and its influence on trim formation is discussed.

Analysis

The undamped, small-angle equations of missile angular motion in classical Euler coordinates, for only a linear static moment and constant roll rate, are[1,3]

$$\ddot\theta+(\omega^2+\mu p\dot\psi-\dot\psi^2)\theta=0 \tag{1}$$

$$\frac{d}{dt}(\dot\psi\theta)+\dot\theta\dot\psi-\mu p\dot\theta=0 \tag{2}$$

$$p=\dot\phi+\dot\psi=\text{const} \tag{3}$$

in which ω^2, the square of the aerodynamic pitch frequency, has been substituted for the ratio of the static moment derivative to the pitch moment of inertia. In the subsequent analysis, ω is assumed to be constant.

In terms of $\dot\Psi$ and Ω, defined in the Nomenclature, Eqs. (1) and (2) can be written

$$\ddot\theta+(\Omega^2-\dot\Psi^2)\theta=0 \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\dot{\Psi}\theta^2)=0, \qquad \dot{\Psi}\theta^2=K \tag{5}$$

We can integrate Eq. (4) by multiplying by $2\dot{\theta}$ and making use of Eq. (5), which yields

$$\dot{\theta}^2+\Omega^2\theta^2+(\dot{\Psi}\theta)^2=E \tag{6}$$

If we substitute for $\dot{\Psi}$ from Eq. (5), we can write Eq. (6) in the form

$$\left[\frac{\mathrm{d}}{\mathrm{d}t}(\theta^2)\right]^2+4(\Omega^2\theta^2-E)\theta^2+4K^2=0 \tag{7}$$

which has the solution

$$\theta^2=a+b\sin(2\Omega t+\tau) \tag{8}$$

The constants a and b are defined in terms of E and K and are related to the maximum and minimum values of θ according to

$$\theta^2_{\max}=a+b, \qquad \theta^2_{\min}=a-b \tag{9}$$

$$\theta_{max}\theta_{\min}=(a^2-b^2)^{1/2}=\pm K/\Omega \tag{10}$$

$$\theta^2_{\max}+\theta^2_{\min}=2a=E/\Omega^2 \tag{11}$$

The sign of K is determined from the sign of $\dot{\Psi}$ (the precession rate direction) in Eq. (5). The appropriate sign multiplying K in Eq. (10) is selected to maintain the right-hand side of Eq. (10) positive, since θ is positive by definition. The corresponding windward-meridian rotation rate $\dot{\phi}$ follows from its definition, Eq. (3), with Eqs. (5) and (10)

$$\dot{\phi}=p-p_r\mp\frac{\Omega\theta_{\max}\theta_{\min}}{\theta^2} \tag{12}$$

The variation of $\dot{\phi}$ with oscillations in θ is shown in Figs. (1) and (2) for both positive and negative precession. For negative precession (Fig. 1), $\dot{\phi}$ is minimum at $\theta=\theta_{\max}$. For positive precession, $\dot{\phi}$ has its minimum magnitude at $\theta=\theta_{\max}$, provided that $\pi(1-\mu/2)<\Omega\theta_{\min}/\theta_{\max}$. We will assume that this inequality exists, so that the minimum rotation rate of the vehicle relative to the wind occurs at $\theta_{\max}$ for both positive and negative precession.

We can calculate the locus of such minima, i.e., the values of windward meridian ϕ about the vehicle at the points of maximum θ and minimum $|\dot{\phi}|$, by integrating Eq. (12) for $\dot{\phi}$. If we denote these points ϕ_1, ϕ_2, ϕ_3, etc., and let $\phi_1=0$ at $t=0$ when $\theta=\theta_{\max}$ and $\dot{\phi}=|\dot{\phi}|_{\min}$, then $\tau=\pi/2$ in Eq. (8) and ϕ_2 is given by

$$\phi_2=(p-p_r)\frac{\pi}{\Omega}\mp\Omega\theta_{\max}\theta_{\min}\int_0^{\pi/\Omega}\frac{\mathrm{d}t}{a+b\cos 2\Omega t} \tag{13}$$

which has the value

$$\phi_2=(p-p_r)\frac{\pi}{\Omega}\mp\pi \tag{14}$$

Similarly, the windward meridian ϕ_3 at the next maximum of θ and minimum of $|\dot{\phi}|$ is

$$\phi_3-\phi_2=(p-p_r)\frac{\pi}{\Omega}\mp\pi \tag{15}$$

and the increment in ϕ between successive points of maximum θ and minimum $|\dot{\phi}|$ is the same as in Eq. (15). The windward-meridian rotation rate $\dot{\phi}$ is the rate at which either the vehicle rotates relative to the wind vector or the wind vector rotates relative to the vehicle. If we consider the wind vector to be rotating about the vehicle, then the successive points ϕ_1, ϕ_2, ϕ_3, etc., at which this rotation rate is minimum and θ is maximum are equally spaced at an angle given by Eq. (15). However, the vehicle rolls clockwise in space at an amount $p\pi/\Omega$ between each successive maximum in θ because the period between successive $\theta_{\max}$ is $\Delta t=\pi/\Omega$, from Eq. (8). Hence, the positions in space of the windward meridians ϕ_1, ϕ_2, ϕ_3, etc., or the locus of minimum windward-meridian rotation rates, is given from Eq. (3) by

$$\Delta\psi=p\Delta t-\Delta\phi=\frac{\pi p_r}{\Omega}\pm\pi \tag{16}$$

and alternates between 0 and π for each successive $|\dot{\phi}|_{\min}$, except for the small angle $\Delta\phi_p=\pi p_r/\Omega$, as shown in Fig. 3.

Application to Asymmetry Formation

If it is postulated that ablation or erosion occurs preferentially at points of minimum $|\dot{\phi}|$, then incipient shape asymmetry would be expected to occur at points about the vehicle that alternate approximately 180 deg in space (Fig. 4). The term "incipient" ablation or erosion is used because once the vehicle develops some trim asymmetry, the resulting trim moments must be included. The motion changes from epicyclic to tricyclic[4] and the coupling between angle of attack and windward-meridian rotation rate becomes more complex.

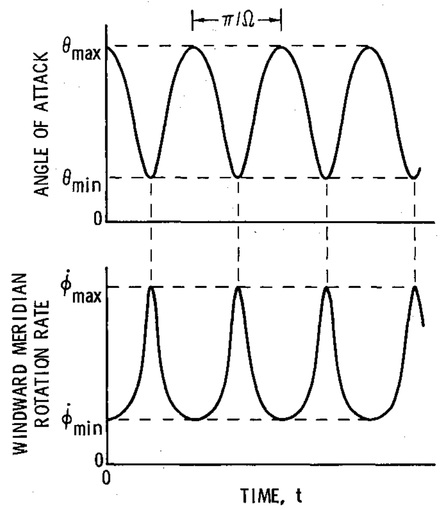

Fig. 1 Coupling between θ and $\dot{\phi}$ for $\dot{\psi}<0$.

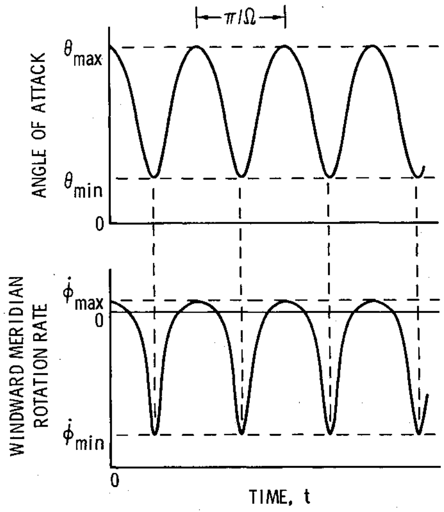

Fig. 2 Coupling between θ and $\dot{\phi}$ for $\dot{\psi}>0$.

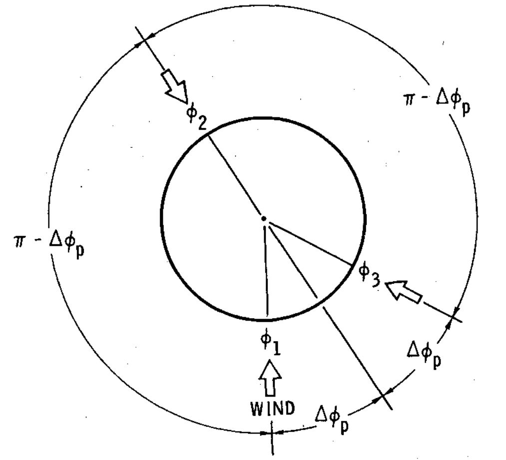

Fig. 3 Locus of minimum windward-meridian rotation rate.

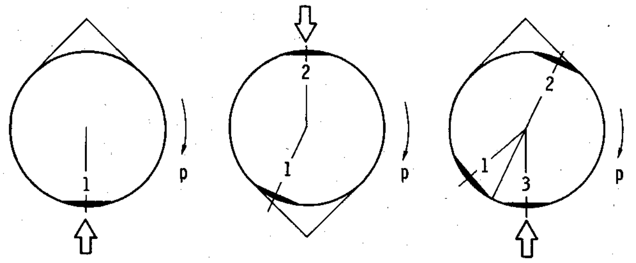

Fig. 4 Incipient asymmetry formation.

The behavior indicated in Figs. 3 and 4 suggests that a small shape change would occur with each pitch oscillation of the vehicle, so that the resulting trim moments would tend to cancel out. If the net trim remained sufficiently small, then the epicyclic motion would persist until it damped out from inherent pitch and normal force damping.[1] Because successive points of minimum $|\phi|$ and presumed shape change occur on essentially opposite meridians (from the point of view of an observer fixed in space), the lift nonaveraging dispersion associated with equal increments of shape change should be minimal.[5,6]

It is emphasized that these conclusions are based on the assumption that very small "incipient" ablation or erosion asymmetries exist which do not influence the motion. A more rigorous treatment of the problem should include the effects of these shape asymmetries on the coupling between the angle of attack and windward-meridian rotation rate.

Acknowledgment

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Measurements of Despin and Yawing Moments Produced by a Viscous Liquid

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Nomenclature

I_A = canister axial moment of inertia
I_0 = fixture moment of inertia (not including canister) about canister coning axis
I_T = canister transverse moment of inertia
M_{F_Ω} = bearing friction moment about canister coning axis
M_{F_ω} = bearing friction moment about canister spin axis
M_{L_Ω} = liquid fill induced yawing moment
M_{L_ω} = liquid fill induced despin moment
XYZ = aeroballistic axes system
θ = canister coning angle
Ω = canister coning rate
ω = canister spin rate
$(\dot{\ })$ = first derivative with time

Introduction

SEVERE flight instabilities, characterized by a sharp increase in projectile yaw angle accompanied by an abrupt loss in spin rate have been experienced by spin-stabilized artillery projectiles having homogeneous, viscous liquid fills.[1] Experimental investigations have shown this flight instability to be produced by motion of the liquid fill and to have a maximum effect for a liquid kinematic viscosity on the order of 100 K cs.[2] Other experimental studies[3,4] indicate that this instability appears to be fundamentally different from the Stewartson type usually associated with projectiles having low viscosity liquid fills. Although several theoretical analyses employing different approaches[5-7] have been completed, additional experimental data are required to fully understand the source of the instability and support the evolution and validation of a comprehensive theoretical basis to describe the phenomena involved. This Note presents laboratory measurements of the liquid fill induced yawing and despin moments produced by the viscous liquid contained in a cylindrical canister undergoing simultaneous spinning and coning motion. The data provide experimental evidence of a direct relation between the destabilizing yawing moment and the despin moment produced by the viscous liquid fill.

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